

Time Warping  
for  
Propagation Analysis

Open Source Hardware Engineering

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When analyzing data related to radio wave propagation, it is sometimes of interest to align the data to coincide with events such as sunrise and sunset. As a case in point, about five months worth of signal readability on the West coast of the U.S. data was available for time signal broadcast by NIST station WWVB on 60kHz. Plotting the data by UTC hour showed a possible correlation of poor reception with sunrise and sunset. The problem is that sunrise and sunset don't occur at the same UTC hour every day. Plotting data versus UTC time will therefore smear out over any correlations when the data covers a period of five months.

As an experiment, each day's worth of reception data was individually shifted so that either sunrise or sunset occurred at the same time. This required calculating the rise or set time for each day and shifting each day's data by a different amount.

This revealed obvious signal dropouts about 70 minutes before sunrise and 20 minutes prior to sunset. This was still not ideal, as the data could only be offset to align with sunrise or sunset events, but not both at the same time.

What's needed is a way to warp time throughout each individual day so that sunrise and sunset always occur at the same point (e.g. sunrise always at 6AM and sunset at 6PM). This warped time will be called *mean time* in what follows<sup>1</sup>. The warping of time should meet these criteria:

- Times half-way between sunrise and sunset should remain the same.
- Sunrise and sunset should map to 6AM and 6PM respectively.
- What happens in between should be smooth.

## Tangents

The answer to this was suggested by the concepts of true and eccentric anomaly from the theory of elliptical orbits in astronomy. There, angles are called anomalies for an unknown reason. Figure 1 shows an object (e.g. a planet),  $P$  orbiting about the focus of an ellipse,  $F$ . The major axis goes through that focus and the center  $C$ , and intersects the perihelion on the right. A circle is circumscribed on the elliptical orbit, and the point  $P'$  is where the vertical line from the major axis through  $P$  intersects this circle.

The *true anomaly* is the angle  $\nu$  between perihelion and  $P$  as seen from  $F$ . The *eccentric anomaly* is the angle from perihelion to  $P'$  as seen from the center  $C$ . These definitions only involve angles, not times or velocities of the orbiting object.

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<sup>1</sup>Our terminology should not be confused with mean anomaly in astronomy; they are entirely different things.

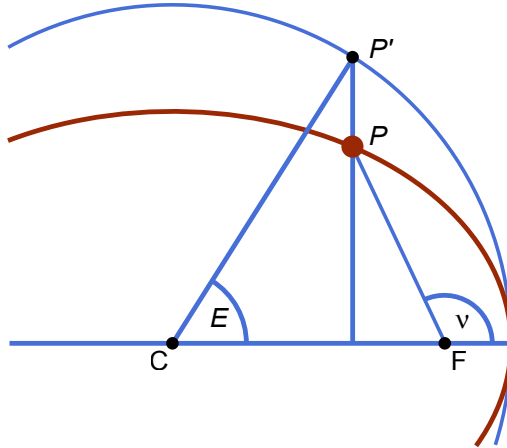


Figure 1: True and eccentric anomaly

The true and eccentric anomalies are equal at perihelion ( $\nu = E = 0$ ) and aphelion ( $\nu = E = \pi$ ). When  $E = \pi/2$  it can be seen that  $\nu$  will be larger than  $E$  by an amount depending the eccentricity of the orbit. This can provide the required mapping if we make the times halfway between sunrise and sunset correspond to the true anomaly at perihelion and aphelion. The mathematical expression for the relationship between anomalies is

$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

Notice that the tangent of one angle is equal to the tangent of the other multiplied by a scale factor. Because of the physical situation represented, the scale factor is always larger than one.

## The Solution

At this point, the analogy to orbital mechanics is dropped and the concept of scaling the tangent of an angle is used to solve the problem at hand. The scale factor will be designated  $\beta$  and it will multiply the tangent of the true angle.

$$\tan \frac{E}{2} = \beta \tan \frac{\nu}{2}, \text{ or } E = 2 \tan^{-1} \left( \beta \tan \frac{\nu}{2} \right) \text{ and } \beta = \frac{\tan E/2}{\tan \nu/2}$$

With this mapping, angles of zero or  $\pi$  are unchanged. What's left is to choose  $\beta$  so that the true angles of sunrise and sunset ( $\nu_r, \nu_s$ ) wind up corresponding to eccentric angles  $E = \pm\pi/2$ . Arbitrarily setting sunset to be

the positive angle, and knowing that it should map to a mean (i.e. eccentric) angle of  $\pi/2$ ,

$$\beta = \frac{\tan(\pi/4)}{\tan(\nu_s/2)} = \cot \frac{\nu_s}{2}$$

## The Details

Usual 24-hour clock time will be called *true time*. It may be expressed in any convenient time zone. The time warped such that sunrise and set always occur at 6AM and 6PM will be called *mean time*. Similarly, angles will either be referred to as being true or mean. In dealing with angles, all values are taken modulo  $2\pi$  such that a negative angle in the range  $\{-\pi..0\}$  is equivalent to an angle in the range  $\{\pi..2\pi\}$ .

Variables in equations will have subscripts of  $r$  and  $s$  to refer to sunrise and sunset respectively. An additional subscript,  $o$  will be added to represent a time that has been offset to a different timezone. Other subscripts should be obvious from the text.

Since times may be in any timezone they must be offset to make sunrise and sunset occur equidistant from both 00:00 and 12:00 – these are equivalent to the angles 0 and  $\pi$ . The first step is to change units from 24-hour clock hours  $T$  to an angle in radians,  $\nu$ .

$$\nu_r = \frac{\pi}{12}T_r, \text{ and } \nu_s = \frac{\pi}{12}T_s$$

Times will now be referred to as angles with this change of units. The next step is to offset the true angles so that sunrise and sunset are of equal magnitude but opposite sign (modulo  $2\pi$ ), with sunset being positive. Since midnight is the hour zero, it will be better to map things so that the angle zero occurs at night instead of during daytime. The time offset will be the angle halfway between sunset and sunrise, and for this we need the duration of night:

$$\delta_{night} = (\nu_r - \nu_s) \pmod{2\pi}$$

Where the mod function is understood to return the smallest positive angle between zero and  $2\pi$ . The halfway point between sunrise and sunset in the original timezone will be called  $\nu_{ref}$ :

$$\nu_{ref} = \left( \nu_s + \frac{\delta_{night}}{2} \right) \pmod{2\pi}$$

Now we reference all times to midnight, and the results are implicitly converted modulo  $2\pi$  here:

$$\nu_{ro} = \nu_r - \nu_{ref}, \text{ and } \nu_{so} = \nu_s - \nu_{ref}$$

These offset true times corresponding to sun rise and set events are now equidistant from angle zero (midnight or  $\nu_{ref}$ ). The scaling factor is now determined from the positive angle (sunrise):

$$\beta = \cot \frac{\nu_{ro}}{2}$$

With these definitions, the mean time for any time of day in the original time zones may now be computed as follows: First, convert the clock time to a true, offset angle,

$$\nu_o = \frac{\pi}{12}T - \nu_{ref} \pmod{2\pi},$$

and find the corresponding mean angle:

$$E = 2 \tan^{-1} \left( \beta \tan \frac{\nu_o}{2} \right),$$

Then convert from mean angle to mean time:

$$T_m = \frac{12}{\pi} E$$

This can all be summarized into this group of equations

$$\nu_r = \frac{\pi}{12}T_r \quad \nu_s = \frac{\pi}{12}T_s$$

$$\delta_{night} = (\nu_r - \nu_s) \pmod{2\pi}$$

$$\nu_{ref} = \left( \nu_s + \frac{\delta_{night}}{2} \right) \pmod{2\pi}$$

$$\nu_{ro} = (\nu_r - \nu_{ref}) \pmod{2\pi}$$

$$\beta = \cot \nu_{ro}$$

$$T_{mean} = \frac{24}{\pi} \tan^{-1} \left( \beta \tan \left[ \frac{\pi T}{24} - \frac{\nu_{ref}}{2} \right] \right)$$

## Tools Attached

There is a zip file attached to this PDF document containing scripts that will run in Matlab or Octave which implement the mean time mapping described

above. This includes functions to calculate sunrise and sunset times. One way to extract the attached zip file is with the `pdftk` utility. Various PDF viewers may also offer the ability to extract attachments.

Using this command line with `pdftk` will unpack the zip file into the same directory as the PDF file.

```
pdftk <input-PDF-filename> unpack_files
```

## An Example

Figure 2 provides a demonstration of the benefits obtained from mapping clock times to mean time. These are histograms showing the distribution of time code reception errors throughout the day across a time span of about five months. Data was taken from February through June in 2019 plus a few days in March of 2020. The Y-axis is a count of the number of minutes for which a valid data frame was not received, and each histogram bin is five minutes wide.

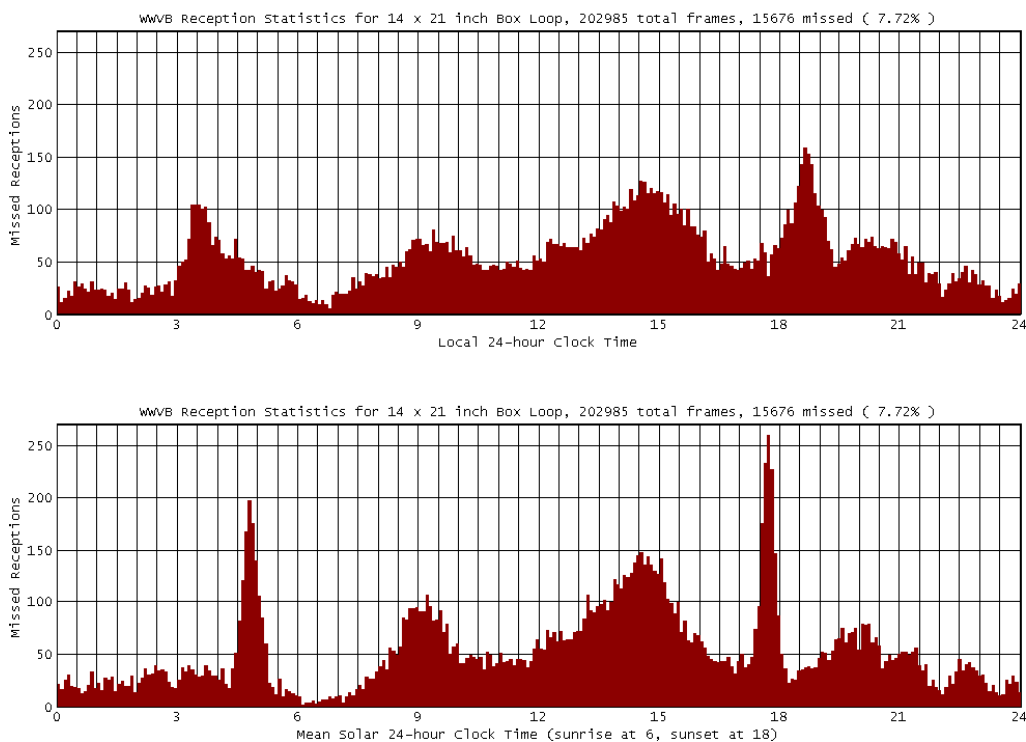


Figure 2: Data viewed in local time versus mean time

In the top graph, data is plotted versus local hour<sup>2</sup>. There are peaks in

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<sup>2</sup>UTC times have been shifted by the longitude of the test location.

the histogram representing a reduction in signal quality at sunrise and sunset and at two other times during the day.

In the bottom graph, the times of missed frames have been converted to mean time, with sunrise and sunset events at exactly 6AM and 6PM. Here, there are very obvious drops in signal quality about 70 minutes prior to sunrise and 20 minutes prior to sunset. These events are much more narrow in time than is apparent from the local time perspective, and more significant than the other two broad peaks during the day.

Reception is significantly improved for a couple of hours after sunrise, and the timing and shape of this is somewhat obscured when viewed w.r.t. local time.

The solar time difference between the test location (U.S. West coast) and the transmitter in Colorado is a little over an hour, and the sunrise event is occurring near the time of sunrise at the transmitter site.